

MU5IN075

Network Analysis and Mining

14. Structure of the Internet

Esteban Bautista-Ruiz, Lionel Tabourier

LIP6 – CNRS and Sorbonne Université

`first_name.last_name@lip6.fr`

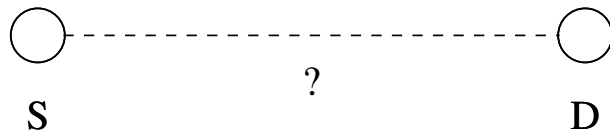
January 11, 2022

Outline

- 1 Introduction: traceroute measurement
- 2 Metrology
 - Influence of sources and destinations
 - Bias on degree
- 3 Correcting the bias

Topology of the internet

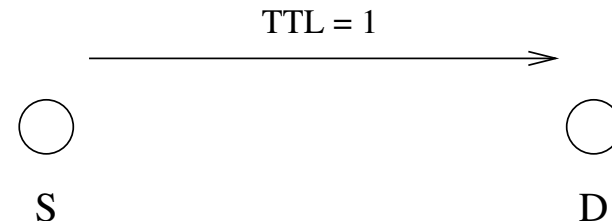
Measurement: exploration using `traceroute`



Principle: packets with same destination and increasing TTL

Topology of the internet

Measurement: exploration using `traceroute`



Principle: packets with same destination and increasing TTL

Topology of the internet

Measurement: exploration using `traceroute`



Principle: packets with same destination and increasing TTL

Topology of the internet

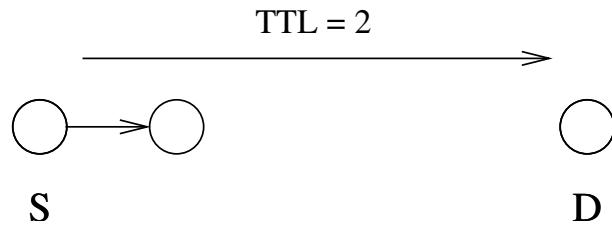
Measurement: exploration using `traceroute`



Principle: packets with same destination and increasing TTL

Topology of the internet

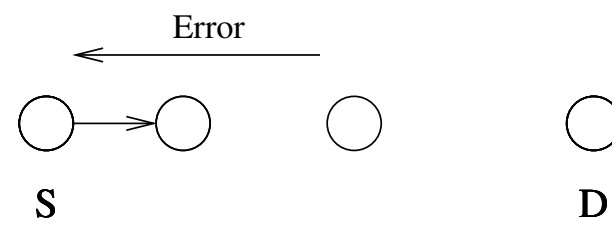
Measurement: exploration using `traceroute`



Principle: packets with same destination and increasing TTL

Topology of the internet

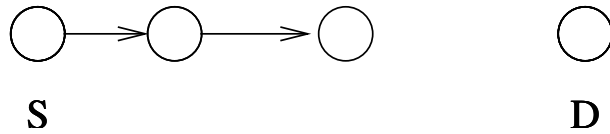
Measurement: exploration using `traceroute`



Principle: packets with same destination and increasing TTL

Topology of the internet

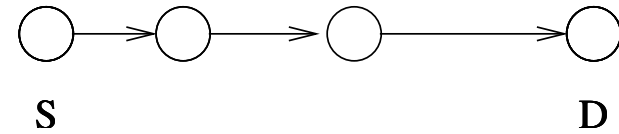
Measurement: exploration using `traceroute`



Principle: packets with same destination and increasing TTL

Topology of the internet

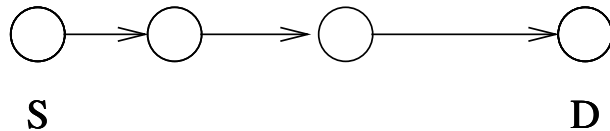
Measurement: exploration using `traceroute`



Principle: packets with same destination and increasing TTL

Topology of the internet

Measurement: exploration using `traceroute`

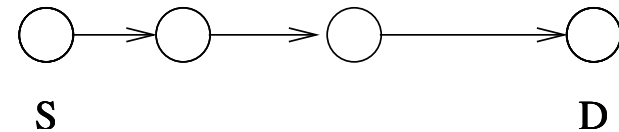


If no answer: *

ICMP filtered for various reasons: *rate limiting, time exceeded,*
...

Topology of the internet

Measurement: exploration using `traceroute`



Remark:

one router = several IP addresses
answers with the IP address that sends the packet
⇒ **simplified description of the process**

Measurement bias

A very general but largely ignored fact about Internet-related measurements is that what we can measure in an Internet-like environment is typically not the same as what we really want to measure (or what we think we actually measure)

Mathematics and the internet: A source of enormous confusion and great potential -
W. Willinger et al., Notices of the AMS, 2009

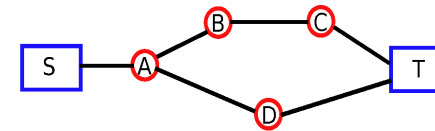
example: traceroute measurement from source S to target T

Measurement bias

A very general but largely ignored fact about Internet-related measurements is that what we can measure in an Internet-like environment is typically not the same as what we really want to measure (or what we think we actually measure)

Mathematics and the internet: A source of enormous confusion and great potential -
W. Willinger et al., Notices of the AMS, 2009

example: traceroute measurement from source S to target T



Problematic

Information collection

Practically, **a few sources, a lot of destinations**

- we know that we don't see everything. . .
- how to get a meaningful view? (→ evaluate **bias**)

Measured property: the degree distribution

Degree distribution of the Internet:
we know it is heterogeneous, close to a **power-law**

On routes and multicast trees in the Internet - Pansiot and Grad, 1998
On power-law relationships of the internet topology - Faloutsos, Faloutsos and Faloutsos, 1999

Problematic

Information collection

Practically, **a few sources, a lot of destinations**

- we know that we don't see everything. . .
- how to get a meaningful view? (→ evaluate **bias**)

Measured property: the degree distribution

Degree distribution of the Internet:
we know it is heterogeneous, close to a **power-law**

On routes and multicast trees in the Internet - Pansiot and Grad, 1998
On power-law relationships of the internet topology - Faloutsos, Faloutsos and Faloutsos, 1999

Surprising degree distribution observed → bias?

How to proceed?

- Experimental: measure from a large number of sources
- Also calls for theoretical studies

Lecture goal
understand and analyze research results
topic: IP maps of the Internet

Surprising degree distribution observed → bias?

How to proceed?

- Experimental: measure from a large number of sources
- Also calls for theoretical studies

Lecture goal
understand and analyze research results
topic: IP maps of the Internet

Outline

- 1 Introduction: traceroute measurement
- 2 Metrology
 - Influence of sources and destinations
 - Bias on degree
- 3 Correcting the bias

Volume of information

On the Marginal Utility of Network Topology Measurements - Barford, Bestavros, Byers, Crovella, 2001

General idea of the article

- Use data from measurements (rather than simulations)
- Evaluate number of nodes/links seen vs number of sources/destinations → unit of the information volume

When using more sources and destinations...

- ... does it increase the volume of information?
- ... does it decrease the bias?

Volume of information

On the Marginal Utility of Network Topology Measurements - *Barford, Bestavros, Byers, Crovella, 2001*

General idea of the article

- Use data from measurements (rather than simulations)
- Evaluate number of nodes/links seen vs number of sources/destinations → **unit of the information volume**

When using more sources and destinations...

- ... does it increase the volume of information?
- ... does it decrease the bias?

8/54

Data

Two datasets

- 8 sources
 - 1277 destinations
 - 1 traceroute every 30 minutes
 - approximately 7 months
-
- 12 sources
 - > 300 000 destinations
 - same measurement method
 - duration unknown

9/54

Data

Remark about the benefit of repeating measurements

Because of **load-balancing**, ...
→ repeating give more information (and more **noise** too...)

9/54

Methodology

Assess the number of nodes seen as a function of

- the number of sources
- the number of destinations

s sources, d destinations → $s \times d$ combination of values

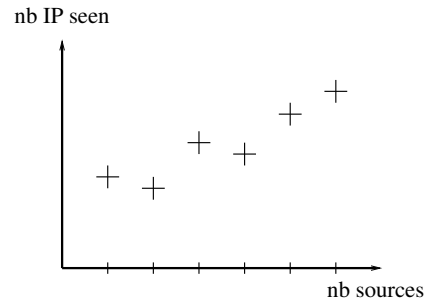
A lot of possibilities...

→ how to choose?

10/54

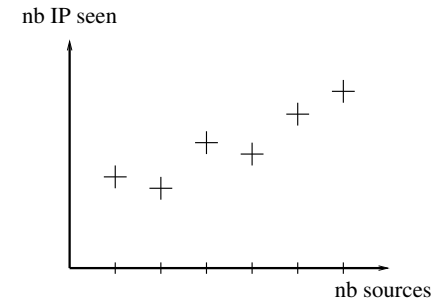
Methodology

What do we want?



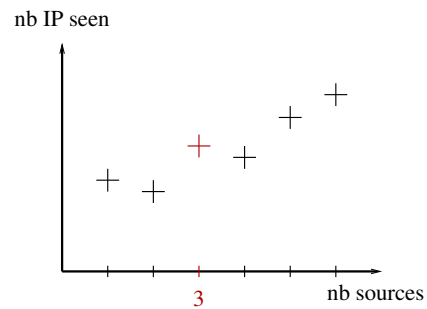
Methodology

What do we want?



same thing with destinations

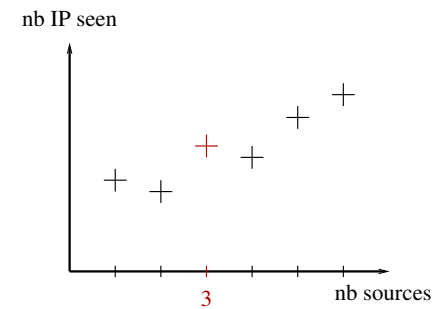
Problem



Number of IPs seen with 3 sources: **which** 3 sources?

Looking for a strategy which maximizes the IPs seen

Problem



Number of IPs seen with 3 sources: **which** 3 sources?

Looking for a strategy which maximizes the IPs seen

Example

One source → set of IPs seen

Example

$s_1 : \{a, b, c, d, e\}$	$s_4 : \{g, h\}$
$s_2 : \{a, b, c, d, f\}$	$s_5 : \{i, j, k\}$
$s_3 : \{a, b\}$	$s_6 : \{a, d\}$

$s_1 + s_3 + s_6 \rightarrow 5$ IPs
 $s_1 + s_4 + s_5 \rightarrow 10$ IPs

Depends on how complementary the sources are
→ no obvious choice

Example

One source → set of IPs seen

Example

$s_1 : \{a, b, c, d, e\}$	$s_4 : \{g, h\}$
$s_2 : \{a, b, c, d, f\}$	$s_5 : \{i, j, k\}$
$s_3 : \{a, b\}$	$s_6 : \{a, d\}$

$s_1 + s_3 + s_6 \rightarrow 5$ IPs
 $s_1 + s_4 + s_5 \rightarrow 10$ IPs

Depends on how complementary the sources are
→ no obvious choice

Example

One source → set of IPs seen

Example

$s_1 : \{a, b, c, d, e\}$	$s_4 : \{g, h\}$
$s_2 : \{a, b, c, d, f\}$	$s_5 : \{i, j, k\}$
$s_3 : \{a, b\}$	$s_6 : \{a, d\}$

$s_1 + s_3 + s_6 \rightarrow 5$ IPs
 $s_1 + s_4 + s_5 \rightarrow 10$ IPs

Depends on how complementary the sources are
→ no obvious choice

Greedy strategy

At each step: add the source which adds most information

Example

$s_1 : \{a, b, c, d, e\}$	$s_4 : \{g, h\}$
$s_2 : \{a, b, c, d, f\}$	$s_5 : \{i, j, k\}$
$s_3 : \{a, b\}$	$s_6 : \{a, d\}$

Greedy strategy

At each step: add the source which **adds most information**

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, c, d, f\}$

$s_3 : \{a, b\}$

$s_4 : \{g, h\}$

$s_5 : \{i, j, k\}$

$s_6 : \{a, d\}$

1 source: s_1

Greedy strategy

At each step: add the source which **adds most information**

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, c, d, f\}$

$s_3 : \{a, b\}$

$s_4 : \{g, h\}$

$s_5 : \{i, j, k\}$

$s_6 : \{a, d\}$

2 sources: $s_1 s_5$

Greedy strategy

At each step: add the source which **adds most information**

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, c, d, f\}$

$s_3 : \{a, b\}$

$s_4 : \{g, h\}$

$s_5 : \{i, j, k\}$

$s_6 : \{a, d\}$

3 sources: $s_1 s_5 s_4$

Greedy strategy

At each step: add the source which **adds most information**

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, c, d, f\}$

$s_3 : \{a, b\}$

$s_4 : \{g, h\}$

$s_5 : \{i, j, k\}$

$s_6 : \{a, d\}$

4 sources: $s_1 s_5 s_4 s_2$

Greedy strategy

At each step: add the source which **adds most information**

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, c, d, f\}$

$s_3 : \{a, b\}$

$s_4 : \{g, h\}$

$s_5 : \{i, j, k\}$

$s_6 : \{a, d\}$

5 sources: $s_1 s_5 s_4 s_2 s_3$

13/54

Greedy strategy

At each step: add the source which **adds most information**

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, c, d, f\}$

$s_3 : \{a, b\}$

$s_4 : \{g, h\}$

$s_5 : \{i, j, k\}$

$s_6 : \{a, d\}$

6 sources: $s_1 s_5 s_4 s_2 s_3 s_6$

13/54

Greedy strategy

At each step: add the source which **adds most information**

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, c, d, f\}$

$s_3 : \{a, b\}$

$s_4 : \{g, h\}$

$s_5 : \{i, j, k\}$

$s_6 : \{a, d\}$

sources: $s_1 s_5 s_4 s_2 s_3 s_6$

Motivation: close to “best” case, without testing all combinations

13/54

Complexity of this processing

Complexity of the union of two sets

Complexity of step 2

compute $n - 1$ unions

(supposing that all sets approximately of the same size for simplicity)

Complexity of step i

compute $n - (i - 1)$ unions

14/54

Complexity of this processing

Complexity of the union of two sets

→ proportional to size of the smallest
(at least, it depends on the implementation)

Complexity of step 2

compute $n - 1$ unions
(supposing that all sets approximately of the same size for simplicity)

Complexity of step i

compute $n - (i - 1)$ unions

Complexity of this processing

Complexity of the union of two sets

→ proportional to size of the smallest
(at least, it depends on the implementation)

Complexity of step 2

compute $n - 1$ unions
(supposing that all sets approximately of the same size for simplicity)
→ $(n - 1) \times k$ if all sets are of size k

Complexity of step i

compute $n - (i - 1)$ unions

Complexity of this processing

Complexity of the union of two sets

→ proportional to size of the smallest
(at least, it depends on the implementation)

Complexity of step 2

compute $n - 1$ unions
(supposing that all sets approximately of the same size for simplicity)
→ $(n - 1) \times k$ if all sets are of size k

Complexity of step i

compute $n - (i - 1)$ unions
→ $(n - i + 1) \times k$

Complexity of this processing

compute $n - (i - 1)$ unions
→ $(n - i + 1) \times k$

so, from step $i = 2$ to step $i = n$
→ $k((n - 1) + (n - 2) + \dots + 2 + 1) = \frac{kn(n-1)}{2}$

total: $O(kn^2)$

long if large number of sources (n)

Complexity of this processing

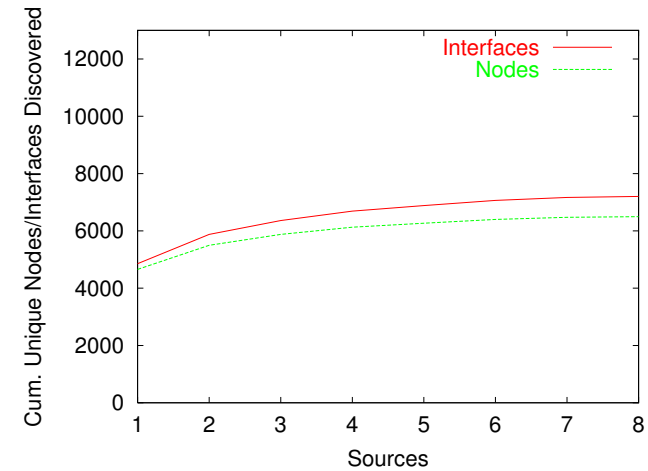
compute $n - (i - 1)$ unions
 $\rightarrow (n - i + 1) \times k$

so, from step $i = 2$ to step $i = n$
 $\rightarrow k((n - 1) + (n - 2) + \dots + 2 + 1) = \frac{kn(n-1)}{2}$

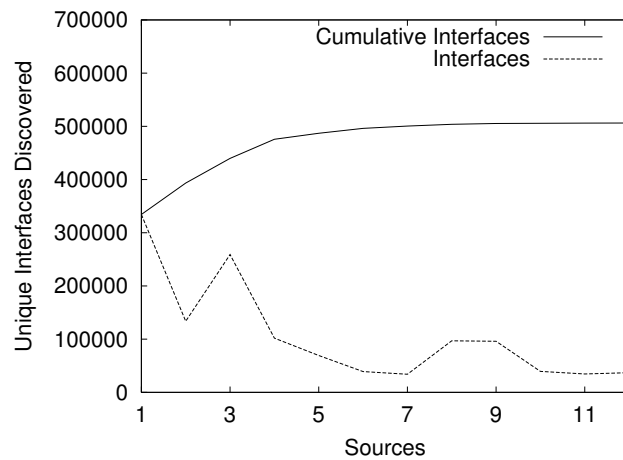
total: $\mathcal{O}(kn^2)$

long if large number of sources (n)

Results



Results



Observations

Convergence of the curve:
the last ones bring nearly no new information
 \rightarrow authors conclude **marginal utility** of source addition

to be discussed later...

Observations

Convergence of the curve:
the last ones bring nearly no new information
→ authors conclude **marginal utility** of source addition

to be discussed later...

Destinations utility

In the ideal case, similar approach:
for every destination → set of IPs seen
but greedy strategy is expensive → **random strategy**

Random strategy

For one source, at each step:

- add randomly a destination

Compare curves for all sources

Destinations utility

In the ideal case, similar approach:
for every destination → set of IPs seen
but greedy strategy is expensive → **random strategy**

Random strategy

For one source, at each step:

- add randomly a destination

Compare curves for all sources

Destinations utility

In the ideal case, similar approach:
for every destination → set of IPs seen
but greedy strategy is expensive → **random strategy**

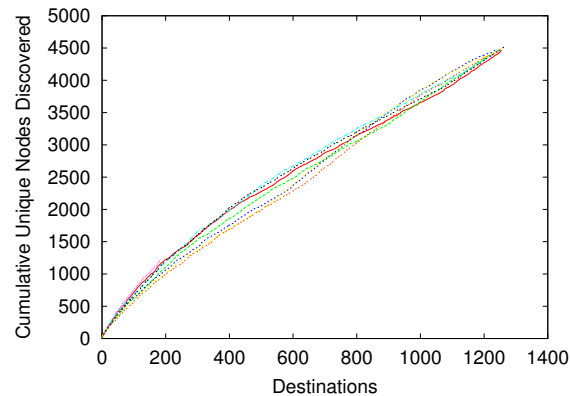
Random strategy

For one source, at each step:

- add randomly a destination

Compare curves for all sources

Results



Observation: roughly linear increase
same benefit for all destinations

Comparison sources and destinations

→ relevant to increase number of destinations rather than number of sources

On the Marginal Utility of Network Topology Measurements - Barford, Bestavros, Byers, Crovella, 2001

We observe difference between curves
→ why such difference between sources and destinations?

while the intuition is
 s sources, d destinations \iff d sources, s destinations

→ importance of the strategy used
greedy vs random

Comparison sources and destinations

→ relevant to increase number of destinations rather than number of sources

On the Marginal Utility of Network Topology Measurements - Barford, Bestavros, Byers, Crovella, 2001

We observe difference between curves
→ why such difference between sources and destinations?

while the intuition is
 s sources, d destinations \iff d sources, s destinations

→ importance of the strategy used
greedy vs random

Comparison sources and destinations

→ relevant to increase number of destinations rather than number of sources

On the Marginal Utility of Network Topology Measurements - Barford, Bestavros, Byers, Crovella, 2001

We observe difference between curves
→ why such difference between sources and destinations?

while the intuition is
 s sources, d destinations \iff d sources, s destinations

→ importance of the strategy used
greedy vs random

Comparison sources and destinations

→ relevant to increase number of destinations rather than number of sources

On the Marginal Utility of Network Topology Measurements - *Barford, Bestavros, Byers, Crovella, 2001*

We observe difference between curves

→ why such difference between sources and destinations?

while the intuition is

s sources, d destinations \iff d sources, s destinations

→ importance of the strategy used
greedy vs random

Critical look at the study

In spite of its interest . . .

Lack of information on

→ the disparity between sources
(one source only sees 184 nodes , > 4000 for the largest one)
→ influence of the strategy implemented

Q: is the choice of sources more important than their number?

Critical look at the study

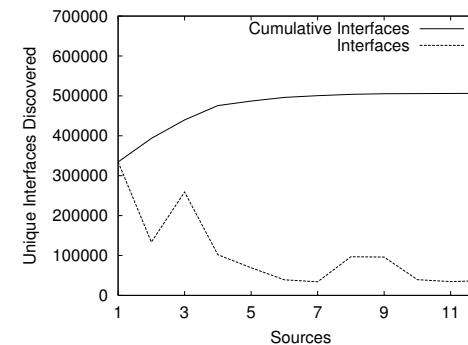
In spite of its interest . . .

Lack of information on

→ the disparity between sources
(one source only sees 184 nodes , > 4000 for the largest one)
→ influence of the strategy implemented

Q: is the choice of sources more important than their number?

Critical look at the study



Last sources: bring few information
but the greedy strategy induces the shape of the curve
no obvious best strategy...

Datasets

To get a better understanding: [compare different strategies](#)

Ouédraogo, Magnien - *Computer Communications*, 2011

Data

- 11 sources
- 3 000 destinations
- 100 traceroutes per day
- ~ 2 months

21/54

Difference between sources

Number of IPs seen per sources

Vary between:

- ~ 16,500
- ~ 26,500

→ Every sources are [not equivalent](#)
(although more homogeneous than in *Barford et al.*)

22/54

Influence of sources and destinations

Three different strategies

- **greedy-max:**
add the source which brings [the most](#) information
- **random:**
add a random source
- **greedy-min:**
add the source which brings [the least](#) information

23/54

Influence of sources and destinations

Greedy strategy \neq maximum possible with k sources

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, e, f\}$

$s_3 : \{a, c, d, g\}$

24/54

Influence of sources and destinations

Greedy strategy \neq maximum possible with k sources

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, e, f\}$

$s_3 : \{a, c, d, g\}$

1 sources : s_1

Influence of sources and destinations

Greedy strategy \neq maximum possible with k sources

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, e, f\}$

$s_3 : \{a, c, d, g\}$

2 sources : $s_1 s_2$

Influence of sources and destinations

Greedy strategy \neq maximum possible with k sources

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, e, f\}$

$s_3 : \{a, c, d, g\}$

3 sources : $s_1 s_2 s_3$

Influence of sources and destinations

Greedy strategy \neq maximum possible with k sources

Example

$s_1 : \{a, b, c, d, e\}$

$s_2 : \{a, b, e, f\}$

$s_3 : \{a, c, d, g\}$

3 sources : $s_1 s_2 s_3$

$s_2 + s_3 : 7$ IPs

$\Rightarrow s_2 + s_3$ gives more info than $s_1 + s_2$

Influence of sources and destinations

Greedy strategy \neq maximum possible with k sources

Example

$s_1 : \{a, b, c, d, e\}$ $s_3 : \{a, c, d, g\}$
 $s_2 : \{a, b, e, f\}$

3 sources : $s_1 s_2 s_3$

$s_2 + s_3 : 7$ IPs
 $\Rightarrow s_2 + s_3$ gives more info than $s_1 + s_2$

Representativeness of maximum? (close to "standard" case?)
Cost to compute the maximum?

Influence of sources and destinations

Other strategies

- Random max \rightarrow **max** over 1000 random orders
- Random min \rightarrow **min** over 1000 random orders
- Random \rightarrow **average** over 1000 random orders

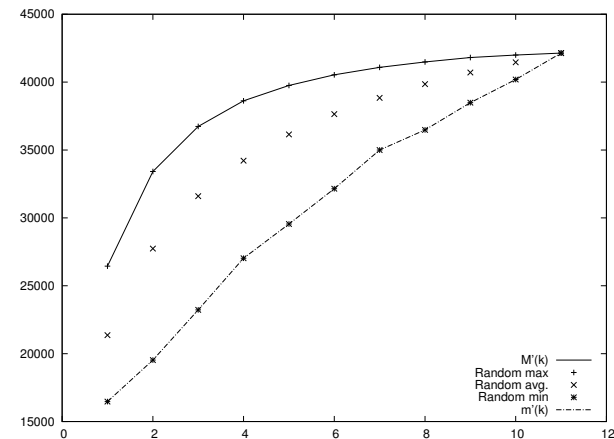
Influence of sources and destinations

Example

$s_1 : \{a, b, c, d, e\}$ $s_4 : \{g, h\}$
 $s_2 : \{a, b, c, d, f\}$ $s_5 : \{i, j, k\}$
 $s_3 : \{a, b\}$ $s_6 : \{i, j\}$

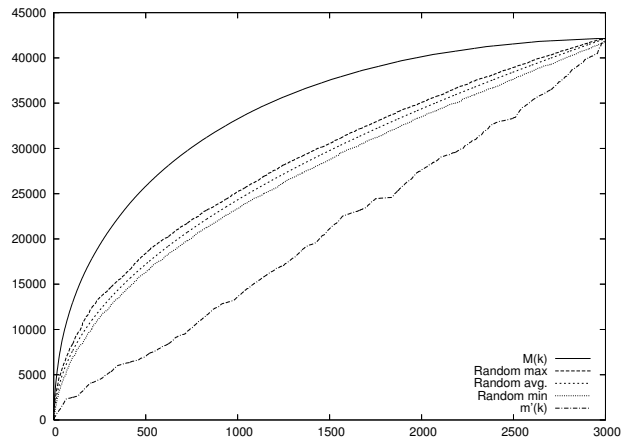
random 1	s_3	s_4	s_6	s_5	s_2	s_1
	2	4	6	7	10	11
random 2	s_5	s_6	s_2	s_4	s_3	s_1
	3	3	7	9	10	11
random min	2	3	6	7	10	11
random max	3	4	7	9	10	11
random average	2.5	3.5	6.5	8	10	11

Results (number of IPs seen)



Influence of **sources**
 rk : M' is greedy-max, m' is greedy-min

Results (number of IPs seen)



Influence of destinations

rk: M' is greedy-max, m' is greedy-min

Observations

- Every curves ends at point n
because every node discovered
- Random max (min) = Greedy max (min) for sources only
because few sources
- Greedy max (averaged)
similar qualitative behaviors for sources and destinations
- In practice, larger variability with sources
because few sources

Observations

- Every curves ends at point n
because every node discovered
- Random max (min) = Greedy max (min) for sources only
because few sources
- Greedy max (averaged)
similar qualitative behaviors for sources and destinations
- In practice, larger variability with sources
because few sources

Observations

- Every curves ends at point n
because every node discovered
- Random max (min) = Greedy max (min) for sources only
because few sources
- Greedy max (averaged)
similar qualitative behaviors for sources and destinations
- In practice, larger variability with sources
because few sources

Observations

- Every curves ends at point **n** because every node discovered
- Random max (min) = Greedy max (min) for sources only because **few sources**
- Greedy max (averaged) **similar qualitative behaviors** for sources and destinations
- In practice, larger variability with sources because **few sources**

28/54

Conclusion

Utility decreases, but not null
→ **choice of sources might be more important than number**

Ouédraogo, Magnien - *Computer Communications*, 2011

29/54

Exploration bias

Sampling Biases in IP Topology Measurements - *Lakhina, Byers, Crovella, Xie, 2003*

Principle of the article: simulation-based

- Generate artificial graphs → topology
- Simulate traceroutes → measure
- Observe and analyze results

Explore the explicative dimension of modelling

30/54

Exploration bias

Sampling Biases in IP Topology Measurements - *Lakhina, Byers, Crovella, Xie, 2003*

Principle of the article: simulation-based

- Generate artificial graphs → topology
- Simulate traceroutes → measure
- Observe and analyze results

Explore the explicative dimension of modelling

30/54

Implementation - graph models

Basic graph models

- Erdős-Rényi
- Fixed degree distribution → configuration model

31/54

Implementation – traceroute simulation

How to simulate traceroute?
... several possibilities

Usual choice

- route = shortest path (not true but default choice)

Shortest path

- One/every shortest paths?
- If one, which one?

32/54

Implementation – traceroute simulation

How to simulate traceroute?
... several possibilities

Usual choice

- route = shortest path (not true but default choice)

Shortest path

- One/every shortest paths?
- If one, which one?

32/54

Implementation – traceroute simulation

How to simulate traceroute?
... several possibilities

Usual choice

- route = shortest path (not true but default choice)

Shortest path

- One/every shortest paths?
- If one, which one?

32/54

The authors' choice

Give a **weight** to each link (\rightarrow **weighted graph**)
 $1 + \epsilon$, with a random $\epsilon \ll 1$

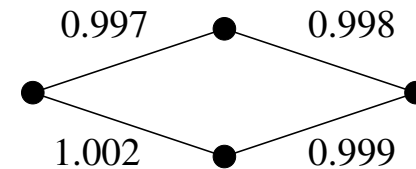
Length of a path: **sum of the weights** of the links
 \rightarrow every paths have different weights

33/54

The authors' choice

Give a **weight** to each link (\rightarrow **weighted graph**)
 $1 + \epsilon$, with a random $\epsilon \ll 1$

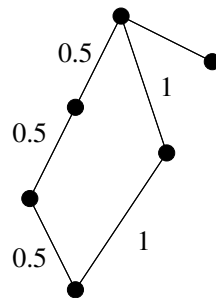
Length of a path: **sum of the weights** of the links
 \rightarrow every paths have different weights



33/54

Computation of the shortest weighted path

Remark: BFS **not suited** for weighted networks



shortest paths from one node in weighted graph (weights>0)
 \rightarrow **Dijkstra** algorithm (not detailed here)

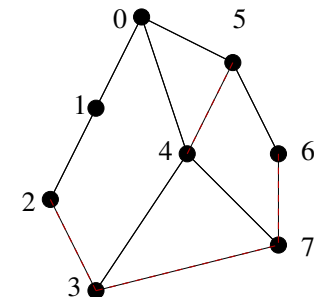
34/54

Our choice: restricted BFS

- no weight
- distances computed with a BFS
- storage of the output of the BFS \rightarrow **table of fathers**

Value **i**: father of **i**
Value **root**: root itself

0	0	1	4	0	0	5	4
0	1	2	3	4	5	6	7



35/54

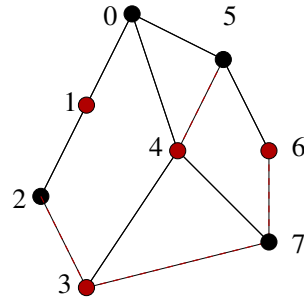
Restiction to destinations

Table initialized at -1

For each destination d : (here : $d = 3, 4, 6, 1$)

- While $AR[d] == -1$
 - $AR[d] = A[d]$
 - $d = A[d]$

A	0	0	1	4	0	0	5	4
	0	1	2	3	4	5	6	7
AR	0	0	-1	4	0	0	5	-1
	0	1	2	3	4	5	6	7



Degree computation

Degree of a node in the BFS tree using table of fathers:

Degree computation

Degree of a node in the BFS tree using table of fathers:

- number of times it appears +1
- except for the **root** : number of times -1

0	0	-1	4	0	0	5	-1
0	1	2	3	4	5	6	7

(boxes with -1: nodes which are not in the BFS tree)

Several sources

Several sources:
 → one BFS **per source**

How to compute the degree of the nodes?
 mark links as **present** or **absent**

Connectedness

Problem if the graph is not connected...

Several solutions

- choose sources and destinations in the same connected component
- use only connected graphs
- ...

No ideal solution

Connectedness

Problem if the graph is not connected...

Several solutions

- choose sources and destinations in the same connected component
- use only connected graphs
- ...

No ideal solution

Connectedness

Problem if the graph is not connected...

Authors' choice:

Restrict to the largest connected component

Simulations

Two cases under study:

Erdős-Rényi graphs (homogeneous degree)

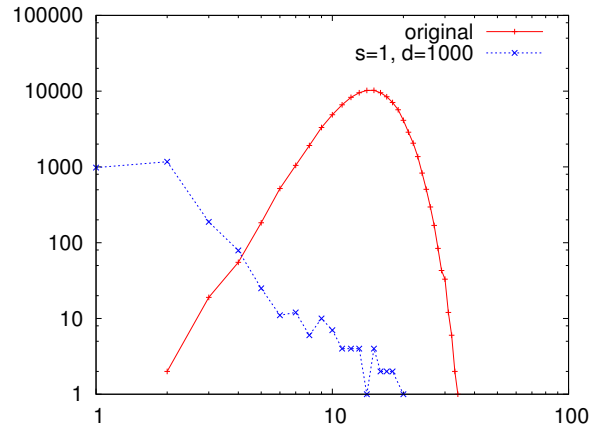
- $n = 100\,000$
- $m = 750\,000$ ($d^\circ(G) = 15$)
- sources: 1, 5, 10
- destinations: 1000, chosen randomly

Fixed degree distribution (heterogeneous degree)

- $n \sim 100\,000$
- $m \sim 190\,000$
- power-law, $\alpha \sim 2.1$

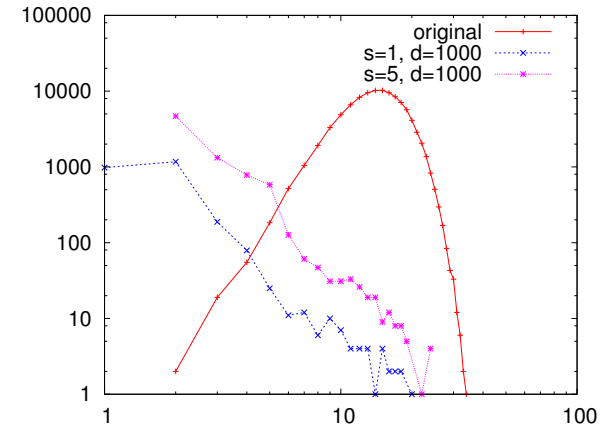
Results

Erdős-Rényi graphs



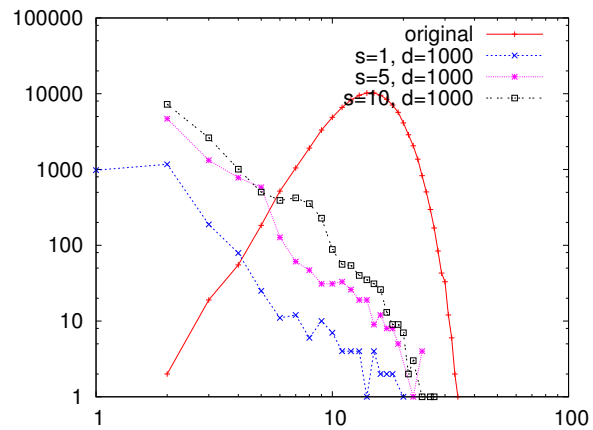
Results

Erdős-Rényi graphs



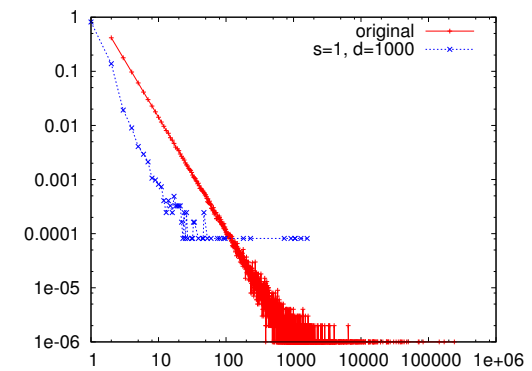
Results

Erdős-Rényi graphs



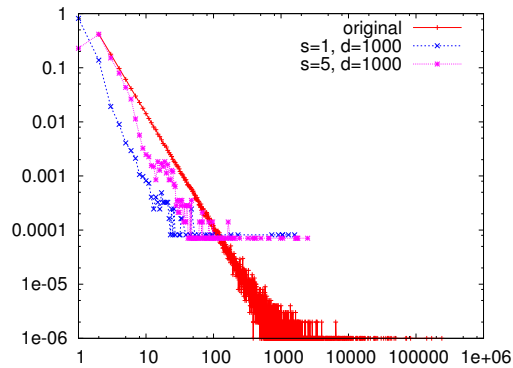
Results

Graphs with fixed heterogeneous degree



Results

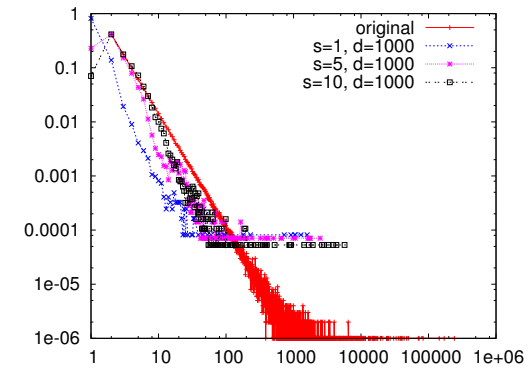
Graphs with fixed heterogeneous degree



42/54

Results

Graphs with fixed heterogeneous degree



42/54

Observations

- Distribution observed \neq real distribution
- Erdős-Rényi: **qualitative** difference
homogeneous appears as **heterogeneous**
- Graphs with fixed degree: **quantitative** difference
slope, max degree, ...

Warning:

ER graphs: Maximum degree observed ~ 30
→power-law models are not reliable at this scale

43/54

Observations

- Distribution observed \neq real distribution
- Erdős-Rényi: **qualitative** difference
homogeneous appears as **heterogeneous**
- Graphs with fixed degree: **quantitative** difference
slope, max degree, ...

Warning:

ER graphs: Maximum degree observed ~ 30
→power-law models are not reliable at this scale

43/54

Conclusion of the study

Observing heterogeneous distrib \Rightarrow Real heterogeneous distrib

No conclusion on the real distribution

44/54

Discussion (1/2)

Important result

- From a theoretical point of view
- Need to be careful about conclusions in practice

What conclusions can we draw from this?

Observed distribution heterogeneous

- is the real distribution homogeneous?
- is the real distribution heterogeneous?

45/54

Discussion (2/2)

Case of ER graphs

Maximal degree observed:

46/54

Discussion (2/2)

Case of ER graphs

Maximal degree observed:
close to **average degree** of the graph.

Experimentally, maximum degree observed > 1000
→ ER graph with average degree $\simeq 1000$? **not realistic**

46/54

Discussion (2/2)

Case of ER graphs

Maximal degree observed:
close to **average degree** of the graph.

Experimentally, maximum degree observed > 1000
→ ER graph with average degree $\simeq 1000$? **not realistic**
→ **real distribution probably heterogeneous...**
need more work

Causes of the bias: first hypothesis

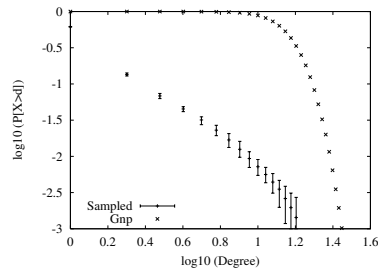
Hyp: Bias in the node sample?

For each node: compare the degree **observed** to its **real** degree

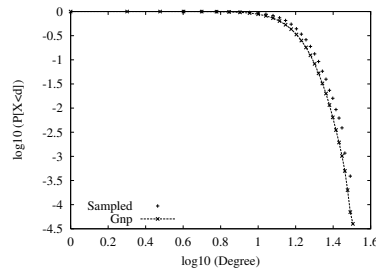
Causes of the bias: first hypothesis

Hyp: Bias in the node sample?

observed deg vs original deg



real deg vs original deg

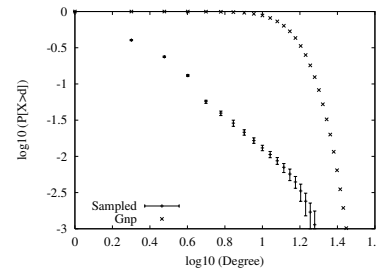


With 1 source

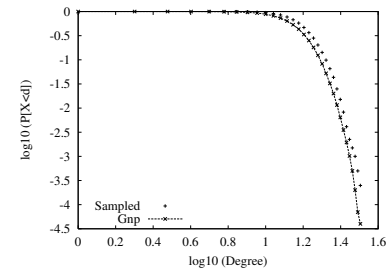
Causes of the bias: first hypothesis

Hyp: Bias in the node sample?

observed deg vs original deg



real deg vs original deg

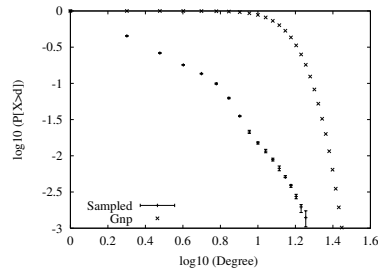


With 5 sources

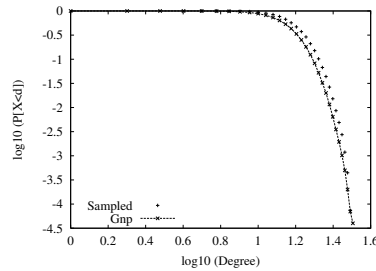
Causes of the bias: first hypothesis

Hyp: Bias in the node sample?

observed deg vs original deg



real deg vs original deg



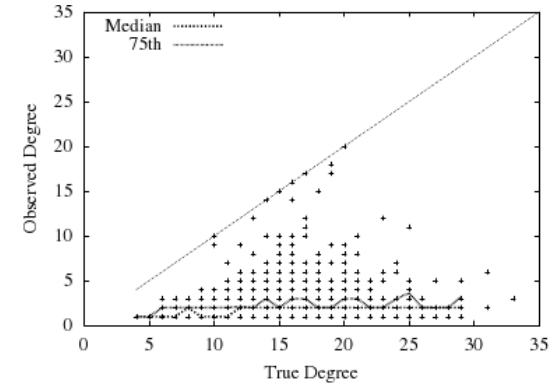
With 10 sources
Nodes are chosen **without bias** on the degree

Causes of the bias: second hypothesis

Hyp: Bias in the link sample?

degree **observed** vs **true** degree

With 1 source

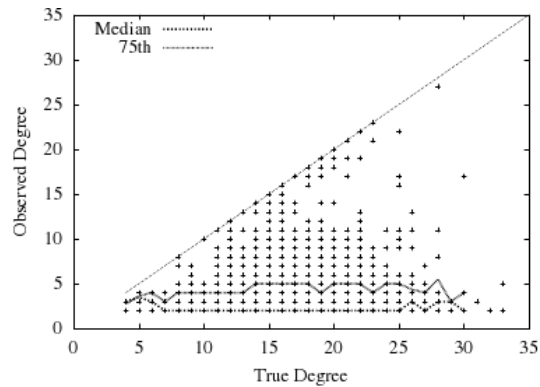


Causes of the bias: second hypothesis

Hyp: Bias in the link sample?

degree **observed** vs **true** degree

With 5 sources

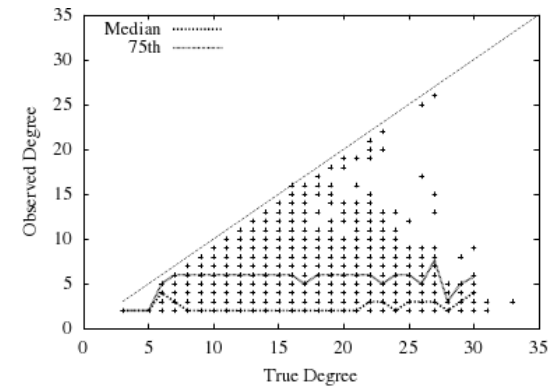


Causes of the bias: second hypothesis

Hyp: Bias in the link sample?

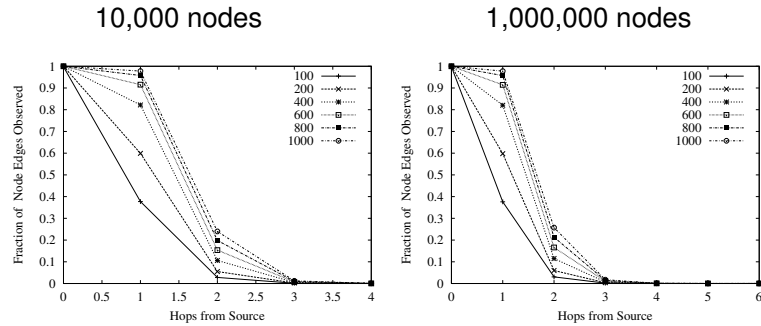
degree **observed** vs **true** degree

With 10 sources



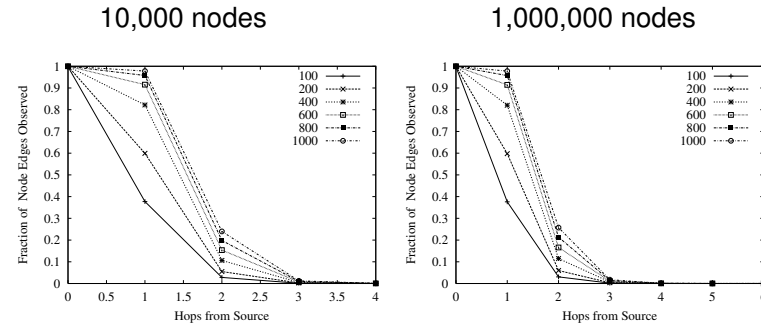
Bias sources

Link visibility as a function of their distance to the source



Bias sources

Link visibility as a function of their distance to the source



The **further** an edge is from the source,
the **less** are its chances to be seen

Outline

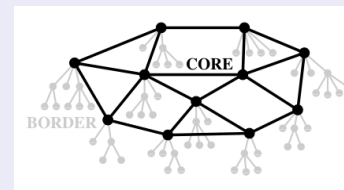
- 1 Introduction: traceroute measurement
- 2 Metrology
 - Influence of sources and destinations
 - Bias on degree
- 3 Correcting the bias

How to make an unbiased measurement?

Rigorous Measurement of the Internet Degree Distribution - *Latapy et al., 2017*

Core routers vs. border routers

- **core**: non-trivial routing
- **border**: routing in a tree

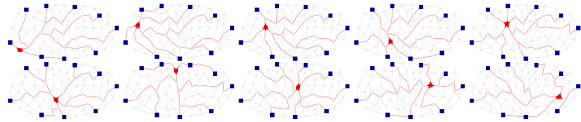


→ focus on core routers degree distribution

Uniform sampling of core routers: protocol

Measurement protocol basis

- 700 sources (PlanetLab monitors)
- 3 million destinations (IP addresses)
- **UDP ping** from all monitors to all destinations on **unallocated port**



Information

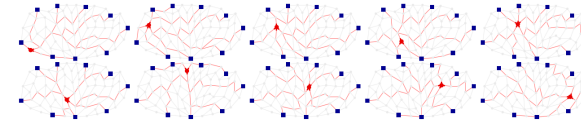
- one of target interfaces sends back error message
⇒ **one interface address of the router**
- enough monitors ⇒ **all core interfaces = router core-degree**

52/54

Uniform sampling of core routers: protocol

Measurement protocol basis

- 700 sources (PlanetLab monitors)
- 3 million destinations (IP addresses)
- **UDP ping** from all monitors to all destinations on **unallocated port**



Information

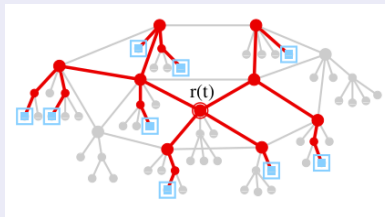
- one of target interfaces sends back error message
⇒ **one interface address of the router**
- enough monitors ⇒ **all core interfaces = router core-degree**

52/54

Unbiased core degree distribution

Nodes filtering

- **core nodes**: all interfaces in the core are seen
- **border nodes**: monitors only detect the interface towards the core



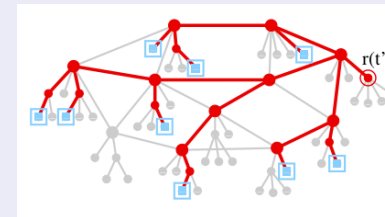
→ 5600 interfaces of reliable core routers

53/54

Unbiased core degree distribution

Nodes filtering

- **core nodes**: all interfaces in the core are seen
- **border nodes**: monitors only detect the interface towards the core



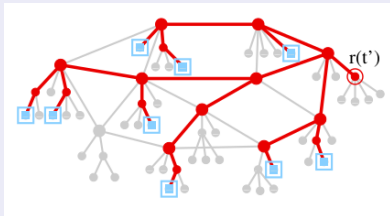
→ 5600 interfaces of reliable core routers

53/54

Unbiased core degree distribution

Nodes filtering

- core nodes: all interfaces in the core are seen
- border nodes: monitors only detect the interface towards the core



→ 5600 interfaces of reliable core routers

Unbiased core degree distribution

During all measurements, we register the interfaces of a reliable router that send an error message

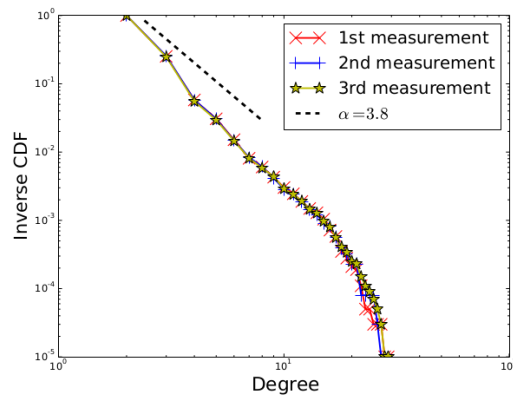
From the observed to the real distribution

Other biases to eliminate:

1. **Discard** border interfaces of core routers
2. **Correct** probability distribution as the probability to sample a router is proportional to its number of interfaces
 - router core-degree k : probability observed p'_k
 - real degree distribution $p_k \propto \frac{p'_k}{k}$

After all that...

(Core) degree distribution of core routers



Heterogeneous, but not a power-law